Performance analysis of a novel uplink cooperative NOMA system with full-duplex relaying

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Abstract: In this study, the authors examine the performance of an uplink non-orthogonal multiple access (NOMA) system with cooperative full-duplex relaying (CFR-NOMA), where the user closer to the base station (BS) is considered as a full-duplex relay to aid the transmission from the other user to the BS. The closer user first decodes the signals transmitted from the far user and then forwards them using superposition coding to the BS on top of transmitting its own information signals to the BS. First, they analyse the performance of the CFR-NOMA system in terms of the outage probability and average sum rate. Then, they analytically obtain the optimal power allocation coefficients that minimise the outage probability. They also study the optimal power allocation between the users to maximise the average sum rate under specified constraints. Their examination demonstrates that the CFR-NOMA system can significantly outperform the uplink conventional orthogonal multiple access and uplink NOMA with half-duplex relaying system in terms of achieving a lower outage probability or a higher average sum rate.

1 Introduction

Wireless communications have achieved great development due to the increasing number of Internet users and other applications, which result in a great amount of wireless connections and tremendous data traffic [1–3]. To accommodate the heavy data traffic, higher spectrum efficiency and data rate are needed in the next generation cellular communications. Towards these targets, non-orthogonal multiple access (NOMA) is considered to be one of the promising candidates for 5G mobile networks [4]. Unlike the conventional orthogonal multiple access (OMA) technique, NOMA allows multiple users to transmit messages at the same time, frequency or space resource simultaneously using different levels of transmit power [5]. At the receiver side, the successive interference cancellation (SIC) [6] will be employed to decode the desired signals sequentially. A downlink NOMA system was discussed in [7], and the achieved system performance of the outage probability and sum rate for randomly deployed users were also investigated. To study the performance of the uplink NOMA scheme, Wei et al. [8] compared the uplink NOMA and OMA communications in terms of the resource allocation fairness. Furthermore, Chen et al. [9] mathematically compared the optimum sum rate performance for NOMA and OMA with the user fairness taken into consideration, and the simulation results showed that NOMA outperforms the OMA scheme significantly.

In order to further enhance the communication reliability of NOMA, the joint applications of cooperative communication and NOMA have attracted great attention [10–12]. The cooperative NOMA scheme was first proposed in [13], where the users in good channel conditions act as relays to strengthen the communication reliability between the users with poor channel conditions and the base station (BS). Similarly, in order to improve the communication reliability for users with worst channel conditions, a cooperative NOMA transmission scheme with a dedicated relay was introduced in [14]. In [15, 16], a cooperative relaying system using NOMA (CRS-NOMA) was studied to further improve the spectral efficiency, and Xu et al. [16] proposed a novel receiver scheme for the CRS-NOMA system by employing maximum-ratio combination (MRC). Also, Men et al. [17] investigated a NOMA-based relaying networks over Nakagami-m fading channels, in which the BS communicates with multiple users simultaneously with an amplify-and-forward relay. Furthermore, Kader et al. [18] investigated a dual-hop cooperative relaying using NOMA and derived the expressions of ergodic sum rate, where two sources communicate with their corresponding destinations in parallel. Besides, the authors in [19–21] studied the cooperative NOMA system with full-duplex (FD) relaying, where the receiver near to the transmitter receives the superimposed signal and meanwhile decodes and forwards the messages to the far receiver. In particular, Zhong and Zhang [19] investigated a FD cooperative NOMA system with dual users, in which a dedicated FD relay was employed to assist the transmission of the user with weak channel condition. The outage probability of both users and average sum capacity were investigated under imperfect SIC technology. In [20], a collaborative NOMA assisted relaying system was introduced, in which three users were served by the BS, and the user nearest to the BS was considered as a relay to establish communications from BS to the other two users. Zhang et al. [21] studied the performance of downlink cooperative NOMA system with FD relaying, where the near user acts as a FD relay to help the far user communicating with BS and there was no direct link that exists between the BS and the far user. Moreover, we have proposed a two-phase superposition coding (SC) scheme for the CRS-NOMA [22], in which the source simultaneously transmits two symbols using SC in the first time slot, while the relay decodes the symbols by employing SIC and then forwards the two symbols with a new SC to the destination, and in the second time slot the destination jointly decodes the signals by employing two-layer MRC technology over the signals transmitted from both the source and relay.

1.1 Motivations and contributions

The aforementioned works established a strong foundation for the performance analysis of downlink cooperative NOMA, while the performance of the cooperative NOMA in uplink has not been well examined. Wei et al. [23] investigated a hybrid downlink–uplink cooperative NOMA scheme, in which the strong user transmits superimposed signal of its own signal for uplink transmission and the decoded signal for the far user during the cooperative phase and all the transceivers are operated in half-duplex (HD) mode. A two-user uplink NOMA system was considered in [24], where the cell-user transmits messages with the help of a dedicated HD relaying.
However, in the uplink two-user NOMA scenario, there is no work in the literature considering the near user as a DF relaying to assist the transmission of the far user. Therefore, we propose an uplink NOMA system with cooperative FD relaying (CFR-NOMA) in this paper, where the near user works as a FD relaying to improve the communication quality of the far user. The proposed uplink CFR-NOMA scheme effectively enhances the transmission reliability of the far user and greatly improves the fairness of the far user. In the uplink CFR-NOMA system, the user far from the BS transmits signals to the BS and the near user simultaneously, while the near user decodes the signal transmitted from the far user and then forwards its own signal and the decoded signal with SC to the BS. The main contributions of this work are summarised as follows:

(i) We first design and analyse an uplink NOMA system with FD relaying, in which the near user works as a FD relay to improve the transmission reliability from the far user to the BS. This is the first time that the cooperative NOMA system is examined in the context of uplink communication systems.

(ii) The outage probability and the average sum rate of this uplink NOMA system are investigated and closed-form expressions for them are derived. Based on our analysis, we achieve the optimal power allocation coefficients that minimise the outage probability of the uplink CFR-NOMA scheme in closed-form expressions. In addition, the optimal power allocation that maximises the average sum rate for the uplink CFR-NOMA system is also examined.

(iii) Thorough numerical results are provided to demonstrate the performance gain achieved by the proposed uplink CFR-NOMA scheme, relative to the uplink conventional OMA and cooperative NOMA with HD relaying (CHR-NOMA) schemes, which are adopted as the benchmark schemes.

1.2 Related works

In the following, we discuss the related works and clarify our motivations. A similar cooperative NOMA system with FD relaying was studied in [10, 21, 25]. Our work differs from these works in the following aspects. The first aspect is on the system settings. Specifically, the authors in [10, 21, 25] investigated the downlink NOMA system, where the BS transmits signals to the far user with the aid of the near user and directly to the near user. In this work, we consider the uplink NOMA communication system, where the far user transmits signals to the BS with the aid of the near user (acting as a relay), while the near user directly transmits the signal from the far user to the BS. The second aspect is on the channel model of User1 to User2. The authors in [10] assumed a two-step (DF) strategy, where User1 sends signal to User2 and User2 sends signal to the BS. In this work, we consider the wireless link from User1 to BS and User2 to BS, respectively. Thus, our system model can be described as follows.

\[
\begin{align*}
    y_1 &= h_1s_1 + h_2s_2 + n_1, \\
    y_2 &= h_3s_1 + h_4s_2 + n_2,
\end{align*}
\]

where \(y_1\) and \(y_2\) denote the received signal at the BS by User1 and User2, respectively, \(h_1\) and \(h_2\) denote the channel coefficients of User1 to User2 and BS to User2, respectively, \(s_1\) and \(s_2\) denote the transmitted signals from User1 and User2, respectively, and \(n_1\) and \(n_2\) are additive white Gaussian noise (AWGN) at User2 with zero mean and variance \(\sigma^2\).

At time \(t\), according to [27], the received signal at User2 is given by

\[
    y_2 = h_3\sqrt{P_1}s_1[t] + h_4\sqrt{P_2}s_2[t] + n_2[t],
\]

where \(x_1\) is the signal transmitted from User1, \(x_2\) denotes the self-interference channel of User2 due to FD relaying. All these wireless links in the network are assumed to be independent Rayleigh fading channels, which experience independent Rayleigh random variables with variances \(\rho_{h_1, h_2, h_3, h_4}\). Let \(h_{1, 2}, h_{3, 4}\) denote the channel coefficients of User1 to User2 (U1–U2) link, User1 to BS (U1–BS) link and User2 to BS (U2–BS) link, respectively.

The related works and studies of the system model are summarised as follows:

(i) We first design and analyse an uplink NOMA system with FD relaying, in which the near user works as a FD relay to improve the transmission reliability from the far user to the BS. This is the first time that the cooperative NOMA system is examined in the context of uplink communication systems.

(ii) The outage probability and the average sum rate of this uplink NOMA system are investigated and closed-form expressions for them are derived. Based on our analysis, we achieve the optimal power allocation coefficients that minimise the outage probability of the uplink CFR-NOMA scheme in closed-form expressions. In addition, the optimal power allocation that maximises the average sum rate for the uplink CFR-NOMA system is also examined.

(iii) Thorough numerical results are provided to demonstrate the performance gain achieved by the proposed uplink CFR-NOMA scheme, relative to the uplink conventional OMA and cooperative NOMA with HD relaying (CHR-NOMA) schemes, which are adopted as the benchmark schemes.
obtain the signal-to-interference plus noise ratio (SINR) for User2 to
decode $x_1$ as

$$\gamma_{1,i} = \frac{p_i \sqrt{P_i}}{\rho_i \sigma_i + \sigma^2} = \frac{\lambda_1 \rho_1}{\rho_i \rho_1 + 1},$$

where $\rho_i = (P_i/\sigma^2)$ and $\rho_1 = (P_1/\sigma^2)$ denote the transmit SNRs at
User1 and User2, respectively.

According to the NOMA protocol, User2 forwards the superimposed signal of the symbols $x_1$ and $x_2$ to the BS, where $x_1$ is the
information signal from User1 that is decoded at User2 and $x_2$ is
User2's own information signal. We note that a processing delay $\tau$
results from the signal processing at User2 (e.g. decoding $x_1$),
which operates in the FD mode. Following [10, 13], this work
assumes that the processing delay $\tau$ is negligible relative to the
total transmission duration, while the two signals from User1 and
User2 are fully resolvable at the BS due to the involved delay $\tau$.

The BS receives uplink signals transmitted from User1 and
User2. The received signal for the direct link at the BS is given by

$$y_i^j = h_i^j \sqrt{P_i} x_i[t] + n_i[t].$$

where $n_i$ denotes the AWGN at the BS with zero mean and
variance $\sigma^2$. Then, the SNR for the BS to decode $x_i$ from the
direct link is given by

$$\gamma_i = \frac{h_i \sqrt{P_i}}{\sigma} = \lambda_i \rho_i.$$  

Meanwhile, the received signal at BS from the cooperative link can
be expressed as

$$y_i^c = h_i^c (\sqrt{P_i} x_i[t - \tau] + \sqrt{P_i} x_2[t - \tau]) + n_i[t].$$

where $a_1$ and $a_2$ represent power allocation coefficients with
$a_1 + a_2 = 1$ at User2. Based on the principle of the SIC-based
NOMA, the BS first decodes the symbol $x_1$ by treating $x_2$ as
interference. To this end, the MRC is adopted at the BS to combine
the received signals $y_i^j$ and $y_i^c$ for decoding $x_1$ [16, 27].
Then the interference caused by the symbol $x_2$ will be subtracted from
the received signals with SIC for decoding the symbol $x_1$. Therefore,
the SINR for decoding $x_1$ and the SNR for decoding $x_2$ are,respectively, given by

$$\gamma_i^{MRC} = \frac{\gamma_i^j + \gamma_i^c}{\lambda_i \rho_i} = \lambda_i \rho_i + \frac{a_1 \lambda_1 \rho_1}{a_2 \lambda_2 \rho_2 + 1},$$  

where $\gamma_i^{MRC} = (a_2 \lambda_2 \rho_2 + 1)$ denotes the SINR at the BS
for decoding $x_1$ from the cooperative link.

3 Performance analysis

In this section, we will analyse system performance in terms of
the outage probability and maximum sum rate to characterise
the superiority of the uplink CFR-NOMA system. We also propose
the optimal power allocation schemes by achieving the minimum
outage probability and maximum sum rate, respectively.

3.1 Outage probability

In this section, we will analyse the outage performance to
characterise the superiority of the uplink CFR-NOMA system.
Outage probability represents the probability of an event that the
achievable rate falls below a given target rate, which is a
good metric for QoS in the system design. We also propose
the optimal power allocation scheme by minimising the outage probability.

First, we define $R_{1,T}$ and $R_{2,T}$ as the target rates to decode
User1's signal $x_1$ and User2's signal $x_2$. Outage event
for the CFR-NOMA system occurs when the BS fails to decode $x_1$ and
$x_2$. Hence, we denote $\gamma_{1,T}$ and $\gamma_{2,T}$ as the target SNRs to decode $x_1$ and
$x_2$ with $\gamma_{1,T} = 2^{R_{1,T}} - 1$ and $\gamma_{2,T} = 2^{R_{2,T}} - 1$, respectively.

The complementary events of the outage for the uplink CFR-
NOMA can be explained as follows: to successfully decode the
symbol $x_i$, one condition shall be met: the BS can decode the $x_i$
successfully, i.e. $\gamma_{MRC} > \gamma_i$. On the other hand, according to
the uplink NOMA protocol [28], there are two conditions should be
met to detect $x_i$ successfully at the BS: the first one is that the BS
can decode the $x_i$ from the superimposed signal which is transmitted
by User2, i.e. $\gamma_i^c > \gamma_i$, and the second one is that the BS
could decode the symbol $x_i$ successfully, i.e. $\gamma_i > \gamma_i^c$, furthermore,
to satisfy the first condition, another condition which is
that the User2 can decode $x_i$ successfully should be met in the
CFR-NOMA system, i.e. $\gamma_i^c > \gamma_i$. From the above analysis, the outage probability of the CFR-NOMA system can be expressed as follows:

$$P_{out} = 1 - \Pr \{ \gamma_{MRC} > \gamma_{1,T}; \gamma_{2,T}; \gamma_{1,T}^c > \gamma_{1,T}; \gamma_{2,T}^c > \gamma_{2,T} \}.\quad (8)$$

The following theorem provides the expression of the outage
probability for the proposed uplink CFR-NOMA system.

**Theorem 1:** The closed-form expression of the outage probability for the CFR-NOMA system is expressed as

$$P_{out} = \begin{cases} 1, & a_1 < 0, \\ 1 - \frac{\rho_1 \beta_2}{\rho_1 \beta_2 + \gamma_{1,T}} \frac{e^{-\mu_2 \gamma_{1,T} + \mu_1 \gamma_{2,T} + \mu_2} - 1}{\mu_1 + \mu_2 + \mu_2}, & 0 < a_1 < \frac{1}{\gamma_{1,T} + 1} \\ 1, & \frac{1}{\gamma_{1,T} + 1} < a_1 \leq 1 \end{cases} \quad (9)$$

where $\mu_1 = (1/\rho_1 \beta_2)$ and $\varepsilon = \max ((\gamma_{1,T} + \gamma_{2,T} + \rho_1 \beta_2)/a_1)$. Based
on the analytical results in (9), when the transmission SNRs $\rho_1$ and $\rho_2$ approach infinity, we can obtain the asymptotic outage probability as follows:

$$P_{out}^{\infty} = \lim_{\rho_1, \rho_2 \to \infty} P_{out} = 1 - \frac{\beta_2}{\rho_1 \beta_2 + \gamma_{1,T} \gamma_{2,T}}. \quad (10)$$

From (10), it is obvious that when $\rho_1$ and $\rho_2$ tend to infinity, the outage probability tends to a constant when the target rate of $x_i$ is fixed.

3.2 Average sum rate

In this subsection, we focus on the average sum rate of the
proposed uplink CFR-NOMA system. The average sum rate $R_{sum}$
is the summation of the average rate $R_i$ of $x_i$ and the average rate $R_2$
of $x_2$. This performance metric is for an adaptive transmission
scheme, where the transmission rates of $x_1$ and $x_2$ are adaptively
chosen. Now we present the analysis of $R_i$ and $R_2$.

3.2.1 Average rate of $x_i$, $R_i$ Based on the fact that the end-to-end
transmission rate of DF relaying is dominated by the weakest
link [29], the achievable rate of $x_i$ is given by

$$R_i = E(\log(1 + \min(\gamma_i, \gamma_{i,MRC}))) \quad (11)$$

For convenience, we let

$$\theta = \min(\gamma_i, \gamma_{i,MRC}) = \min((\lambda_i \rho_i)/(\lambda_2 \rho_2 + 1), \lambda_i \rho_i + (a_1 \lambda_1 \rho_1)/(a_2 \lambda_2 \rho_2 + 1)).$$
where \( f_\theta(\theta) \) and \( F_\theta(\theta) \) denote the probability density function (PDF) and cumulative distribution function (CDF) of random variable \( \theta \), respectively. As such, \( F_\theta(\theta) \) can be formulated as

\[
F_\theta(\theta) = \Pr\left\{ \min\left\{ \frac{\lambda_1 \phi \mu_1}{\lambda_2 \rho_2 + 1}, \phi \mu_1 + \frac{a_2 \phi \mu_2}{\lambda_2 \rho_2 + 1} \right\} < \theta \}
= 1 - \Pr\left\{ \frac{\lambda_2 \rho_2}{\lambda_2 \rho_2 + 1} > \theta, \phi \mu_1 + \frac{a_2 \phi \mu_2}{\lambda_2 \rho_2 + 1} > \theta \right\}.
\]

The following theorem provides the expression of the achievable rate of 5G, 2018, Vol. 12 Iss. 19, pp. 2408-2417.

**Theorem 2:** The closed-form expression for the achievable rate of \( x_i \) for the uplink CFR-NOMA system.

\[
R_i = \mathcal{E}\left( \ln \frac{\mu_i}{a_i} \right) - \mathcal{E}(0) - \mathcal{E}(0) \left( \frac{a_i \mu_i}{\lambda_i} - \beta_i \right)
+ \frac{\phi e^{\phi \mu \lambda} - \mathcal{E}(0) \mathcal{E}(0)}{\phi e^{(\phi \mu \lambda) + \mu_2}} \left( \frac{a_i \mu_i}{\lambda_i} + \frac{a_i \mu_i}{\lambda_i} \right) + \mathcal{E}(0) \left( \ln \left( \frac{\mu_i}{\lambda_i} \right) \right) + \mathcal{E}(0) \left( \ln \left( \frac{a_i \mu_i}{\lambda_i} \right) \right).
\]

**Proof:** The proof of Theorem 2 is provided in Section 8.2 of Appendix. □

### 3.2.2 Average rate of \( x_i, R_x \): Similarly, the achievable rate of \( x_i \) can be obtained as

\[
R_x = \mathcal{E}(\ln(1 + \gamma_i)) = \mathcal{E}(\ln(1 + \alpha_i \phi \beta_i)).
\]

If we let \( \omega = a_i \phi \beta_i \), (15) can be written as \( R_x = \mathcal{E}(\ln(1 + \omega)) \), Similarly to (12), \( R_x \) can be expressed as

\[
R_x = \frac{1}{\ln c} \int_0^c \frac{1 - F_{\theta}(\omega)}{1 + \omega} d\omega.
\]

Based on the definition of a CDF, we can obtain \( F_{\theta}(\omega) \) as follows:

\[
F_{\theta}(\omega) = \Pr\{a_i \phi \beta_i < \omega\}
= \int_0^\omega \frac{1}{\beta_i} e^{-\omega/\beta_i} d\omega
= 1 - e^{-\omega/\beta_i}.
\]

Thus, the achievable rate \( R_x \) can be calculated as

\[
R_x = \frac{1}{\ln c} \int_0^c \frac{1}{1 + \omega} e^{-\omega/\beta_i} d\omega
= \frac{1}{\ln c} \int_0^c \frac{1}{1 + \omega} d\omega.
\]

### 3.3 Power allocation

In this subsection, we focus on the power allocation issue for the uplink CFR-NOMA system.

### 3.4 Optimal power allocation to minimise the outage probability

In this part, we analyse the optimal power allocation to minimise the outage probability. Note that the efficient power allocation parameter parameters \( a_i \) and \( a_i^* \) at User2 can be determined to minimise the outage probability.

From (9), the outage probability for the CFR-NOMA system holds to be 1 for \( a_i \geq (1/(\gamma_i + \tau)) \), i.e. \( a_i \geq 1/\omega \). Thus, the optimal \( a_i \) must satisfy \( 0 < a_i < (1/(\gamma_i + \mu)) \). In the following, we consider the outage probability \( P_{out} \) in two cases:

Case 1: Both \( 0 < a_i < (1/(\gamma_i + \tau)) \) and \( \gamma_i T/(\gamma_i - T + \gamma_i T - \tau) \) are satisfied. This is equivalent to \( 0 < a_i < (\gamma_i T/(\gamma_i - T + \gamma_i T - \tau)) \). In this case, we have

\[
P_{out} = 1 - \frac{\rho_1 \beta_1}{\rho_1 \beta_1 + \gamma_i T \beta_2} e^{-\gamma_i T / (\rho_1 \beta_1 + \gamma_i T \beta_2)}.
\]

Case 2: Both \( 0 < a_i < (1/(\gamma_i + \tau)) \) and \( \gamma_i T/(\gamma_i - T + \gamma_i T - \tau) \leq a_i < (1/(\gamma_i + \mu)) \). In this case, the outage probability \( P_{out} \) can be expressed as

\[
P_{out} = 1 - \frac{\rho_1 \beta_1}{\rho_1 \beta_1 + \gamma_i T \beta_2} e^{-\gamma_i T / (\rho_1 \beta_1 + \gamma_i T \beta_2)}.
\]

In the following corollary, we find optimal power allocation coefficients \( a_i^* \) and \( a_i^* \) using the closed-form expressions.

**Corollary 1:** The close-form expressions of the optimal power allocation coefficients \( a_i^* \) and \( a_i^* \) are expressed as

\[
a_i^* = \frac{\gamma_i T}{\gamma_i T} \left( 1 + \gamma_i T \right),
\]

\[
a_i^* = \frac{\gamma_i T}{\gamma_i T} \left( 1 + \gamma_i T \right).
\]

**Proof:** The proof of Corollary 1 is provided in Section 8.3 of Appendix. □

### 3.5 Optimal power allocation to maximise the average sum rate

In this part, we study the power allocation to maximise the average sum rate with a specific QoS constraint at User1.

For a power allocation \( (a_{\mu}, a_i) \), the achievable rate of User1 is

\[
R(a_{\mu}, a_i) = \log(1 + \min(\gamma_i T, \gamma_i T, \gamma_i T, \gamma_i T, \gamma_i T, \gamma_i T)).
\]

and the achievable rate of User2 is

\[
R(a_{\mu}, a_i) = \log(1 + \gamma_i T).
\]

According to the above discussion, User1 is to be served for quickly connected transmission, such as incident avoidance alerts and warning messages. Thus, it should be given a higher priority, i.e. we should guarantee its QoS constraint in the considered NOMA system. Then, we formulate the average sum rate maximisation problem as

\[
\max_{a_{\mu}, a_i} R(a_{\mu}, a_i) = R(a_{\mu}, a_i) + R(a_{\mu}, a_i) \quad s.t. \quad R(a_{\mu}, a_i) \geq Q_1,
\]

\[
a_{\mu} + a_i = 1.
\]
where $Q_s$ denotes the minimum data rate required by User1.

**Corollary 2:** The optimal power allocation coefficients for the above optimisation problem satisfy

$$R_*(a_s, a_s') = Q_s.$$  \hspace{1cm} (25)

**Proof:** The proof of Corollary 2 is provided in Appendix 8.4.  

Based on the above analysis, we can adopt a one-dimensional numerical search method to find the optimal power allocation coefficients $a_s'$ and $a_s$ satisfying the following equation set:

$$\begin{align*}
R(a_s', a_s) &= Q_s,  \\
|a_s' + a_s| &= 1.
\end{align*}$$  \hspace{1cm} (26)

**Remark 1:** Complexity analysis (with the uplink conventional OMA and CHR-NOMA schemes as benchmarks): For the uplink CFR-NOMA scheme, User1 transmits signal $x_1$ to the BS and User2. After decoding the signal $x_1$ received from User1, User2 works as a FD relay and forwards a superposed signal to the BS with SC technology. In the receiver side, the BS jointly decodes the signal transmitted via the direct link and the cooperative link by employing MRC and SIC. In the uplink conventional OMA scheme, User1 and User2 communicate with the BS in the TDMA mode, for which SIC, SC or MRC are not required. For the uplink CHR-NOMA scheme, User1 and User2 communicate with the BS in the time division multiple access (TDMA) mode, for which SIC, SC or MRC are not required. For the uplink CFR-NOMA scheme, User2 works as a HD relay to assist the transmission from User1 to the BS. According to the above discussion, the computational complexity of the proposed uplink CFR-NOMA scheme is higher than the uplink OMA system, since MRC and SIC at the receiver side are adopted, while the SC technology is also involved at User2. On the other hand, the uplink CFR-NOMA has a higher complexity than the CHR-NOMA scheme, since in the FD relaying system self-interference cancellation is required. Although the complexity of our proposed uplink CFR-NOMA scheme is higher than the conventional OMA and CHR-NOMA schemes, the CFR-NOMA scheme achieved significant advantages over the OMA and CHR-NOMA schemes, which will be confirmed in our numerical results.

4 Numerical simulations

In this section, we provide the numerical simulation results to validate the performance for the uplink CFR-NOMA system. In order to verify the accuracy of the aforementioned expressions in Section 3, we also provide Mont Carlo-simulated performance results in this section. The distance of all links can be represented by the average power. Particularly, the simulations illustrated below are considered with an normalised distance parameter of the U1–BS link as $\beta_{s1} = 1$. Therefore, we consider the average power of U1–U2 and U2–BS link as $\beta_{s2} = 10$ and $\beta_{s2} = 10$, respectively. Moreover, for simplicity, we set $P_1 = P_2 = \rho$ in this section, thus we obtain the transmission SNR $\rho_1 = \rho_2 = \rho$.

In the following, we compare the proposed CFR-NOMA system with the uplink conventional OMA system and the CHR-NOMA system, respectively.

4.1 Compared with OMA scheme

In this subsection, comparisons are conducted with the uplink conventional OMA system as a benchmark, in which User1 and User2 communicate with the BS in the TDMA mode. Here, we set $E[|h_{s1}|^2] = -20$ dB in the conducted comparisons. In Fig. 2, we plot the average sum rate and the achievable rates of User1 and User2 versus the transmit SNR $\rho$ with different values of $a_s$. The solid and dashed curves represent the average rate for $a_s = 0.1$ and $a_s = 0.3$, respectively. In this figure, we first observe that the theoretical curves precisely match the simulated curves, which demonstrates the the effectiveness of the approximation we employed in equation (4). From this figure, we also observe that the achievable rate of User1 approaches to a constant value in the high-SNR regime, which is due to the fact that the traditional OMA system at low SNR regime, in terms of achieving a lower outage probability. Finally, in this figure we observe that the average sum rate achieved in the CFR-NOMA system is significantly higher than that achieved in the conventional OMA system, since NOMA enables each user to exploit all time resource while OMA system in the TDMA mode limits the time resource that each user can use.

In Fig. 3, we demonstrate the outage behaviour of the CFR-NOMA system by invoking numerical comparisons with the conventional OMA system. In this figure, we present the outage probability with different target rates versus the transmit SNR $\rho$, where the sold curve represents the outage probability for the target rates $R_{1,T} = R_{2,T} = 1$ and $R_{1,T} = R_{2,T} = 1.6$. Finally, in this figure we observe that the analytical and simulated results precisely match, which confirms the correctness of our analysis. In addition, we observe that the CFR-NOMA system significantly outperforms the OMA system at low SNR regime, in terms of achieving a lower outage probability. This is mainly due to the fact that the traditional OMA system in TDMA mode has to double its transmission rates to maintain the same target sum rate as the CFR-NOMA system, which leads to a higher outage probability. Finally, in this figure we can find the outage probability of the CFR-NOMA system is
subject to a performance floor, which is consistent with our analytical result given in (10). This leads to the fact that the OMA system can outperform the CFR-NOMA system in the regime of high transmit SNR. Intuitively, this is due to the fact that the NOMA system is interference limited for User1 (i.e. the weak user) and the outage probability for User1 is limited by a performance floor, which results in that the overall outage probability is limited by the performance floor given in (10).

Fig. 4 presents the outage probability versus the target rate \( R_{1,T} \) with different transmission SNR \( \rho \). We can find that our theoretical derivations are verified by the simulations. Through this figure, we can observe that the uplink CFR-NOMA system gains lower outage probability than the OMA one for different target rate values.

In Fig. 5, we investigate the minimum outage probability with optimal power allocation scheme versus the target rate \( R_{1,T} \) (here we consider \( R_{1,T} = R_{2,T} \) ) with different transmission SNR \( \rho \). We can find that our theoretical derivations are verified by the simulations. Through this figure, we can observe that the uplink CFR-NOMA system gains lower outage probability than the OMA one for different target rate values.

In this subsection, we conduct some comparisons between the considered CFR-NOMA system with the uplink NOMA system with HD relaying (i.e. the CHR-NOMA system). In the uplink CHR-NOMA system, the near user decodes the signal transmitted from the far user and then forwards the superimposed signal of this decoded signal and its own signal to the BS with HD mode. There are two time slots involved in this system, since the HD relay cannot decode and forward signals simultaneously.

In Fig. 6, we plot the average sum rates achieved in the uplink CFR-NOMA and uplink CHR-NOMA systems versus the transmit SNR \( \rho \). In this figure, we first observe that the CFR-NOMA system achieves a higher average sum rate than the CHR-NOMA system with different levels of self-interference. The reason is that the CHR-NOMA system needs two time slots to accomplish one round transmission, while the CFR-NOMA system only requires one time slot. This observation confirms that the factor 1/2 due to the HD relaying in the CHR-NOMA system brings much more negative impacts on the average sum rate than the self-interference due to the FD relaying in the CFR-NOMA system.

In Fig. 7, we plot the outage probability of the uplink NOMA system with FD/HD relaying versus the transmit SNR \( \rho \). We examine the impact of the self-interference on the outage probability of the CFR-NOMA scheme. In this figure, we first observe that the CFR-NOMA scheme outperforms the CHR-NOMA scheme significantly at low-SNR regime. This is due to the fact that interference is not the dominating impact factor on the transmission.
allocation coefficients and the simulation results achieved by the CHR-NOMA system becomes more prominent as the interference values. In this figure, in order to obtain the maximum sum rate, we employed one-dimensional search method to find the optimal power allocation coefficients according to the equation set in (26). From this figure, we can see that maximum sum rate will be greater with higher $\rho$ and smaller self-interference.

5 Conclusion

In this paper, we first propose an CFR-NOMA in the uplink communication systems, where the near user acts as a FD relay for the far user. The outage probability and average sum rate are investigated. We first derived the closed-form analytical expressions on the outage probability and average sum rate, and then we obtain the optimal power allocation coefficients with closed-form expression to minimise the outage probability. Furthermore, we investigate the optimal power allocation scheme to maximise the average sum rate with a specific QoS constraint. Finally, the simulation results have been shown that the CFR-NOMA system significantly improves the performance of the outage probability over the conventional uplink OMA and CHR-NOMA systems.

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7 References

Appendix 8

8.1 Proof of Theorem 1

From (8), we can first calculate
\[
\text{Pr}\{\gamma_{\text{MRC}} > \gamma, \tau > \tau_1, \gamma > \gamma_1, \tau > \tau_2, \gamma > \gamma_2, \tau > \tau_3, \gamma > \gamma_3, \tau > \tau_4, \gamma > \gamma_4, \tau > \tau_5, \gamma > \gamma_5, \tau > \tau_6, \gamma > \gamma_6, \tau > \tau_7, \gamma > \gamma_7, \tau > \tau_8, \gamma > \gamma_8, \tau > \tau_9, \gamma > \gamma_9, \tau > \tau_{10} \} 
\]
for computing the outage probability

\[
\text{Pr}\{\gamma_{\text{MRC}} > \gamma, \tau > \tau_1, \gamma > \gamma_1, \tau > \tau_2, \gamma > \gamma_2, \tau > \gamma_3, \tau > \gamma_4, \tau > \gamma_5, \tau > \gamma_6, \tau > \gamma_7, \tau > \gamma_8, \tau > \gamma_9, \tau > \gamma_{10} \} 
\]

We can observe that if the condition
\[
[\alpha, \beta_2, \gamma_1, \beta_2] = \left[\frac{a\beta_1\beta_2}{\alpha\beta_2} + 1, \frac{a\beta_1\beta_2}{\alpha\beta_2} + 1, \frac{a\beta_1\beta_2}{\alpha\beta_2} + 1 \right] > 0, \tau > \gamma_2, \tau > \gamma_3, \tau > \gamma_4, \tau > \gamma_5, \tau > \gamma_6, \tau > \gamma_7, \tau > \gamma_8, \tau > \gamma_9, \tau > \gamma_{10} \}
\]

We can further be calculated as follows:

\[
\text{Pr}\{\gamma_{\text{MRC}} > \gamma, \tau > \tau_1, \gamma > \gamma_1, \tau > \gamma_2, \gamma > \gamma_3, \tau > \gamma_4, \tau > \gamma_5, \tau > \gamma_6, \tau > \gamma_7, \tau > \gamma_8, \tau > \gamma_9, \tau > \gamma_{10} \} 
\]

According to (28), we can obtain the outage probability

\[
\text{Pr}\{\gamma_{\text{MRC}} > \gamma, \tau > \tau_1, \gamma > \gamma_1, \tau > \gamma_2, \gamma > \gamma_3, \tau > \gamma_4, \tau > \gamma_5, \tau > \gamma_6, \tau > \gamma_7, \tau > \gamma_8, \tau > \gamma_9, \tau > \gamma_{10} \} = 0 \]

Using \( a_1 = a_2 = 1 \), we will have the outage probability \( P_{\text{out}} = 1 \) when \( 1/\gamma_1 + 1 \). Thus, we can calculate the outage probability by assuming the condition \( a_1 < 1/\gamma_1 + 1 \) is satisfied in the following.

When \( a_1 < 1/\gamma_1 + 1 \), (28) can be obtained as (see (29)).

Combining (8) and (29), the outage probability can be obtained and the proof of Theorem 1 is completed.

8.2 Proof of Theorem 2

Using the PDFs \( f_\theta(x) = (1/\theta) e^{-x/\theta} \) \( \theta \in [1, 2, \ldots, 2, 3, 4, \ldots, 1] \), we can obtain \( F_{\theta}(\theta) \) as (see (30)).

From (30), it is very difficult to get the exact expression of \( R \) directly, and thus we consider the scenario with high transmit SIR, in which \( (a\beta_1\beta_2)/(\alpha\beta_2) > 0 \). Accordingly, (30) can be expressed as (see (31)).

Based on (12) and (31), the average rate of \( x \) is derived as (see (32)).

To improve Theorem 2, we first calculate

\[
A_2 = \left\{ (1/\ln 2) \int_{(1+\theta)}^{(1/\ln 2)} f_{\theta}(x) \, dx \right\}
\]

after some partial fraction decompositions, we can obtain (see equation below). Therefore, \( A_2 \) can be solved as follows: (see (33)), where we use \( \int_{x}^{x+1}(x) \, dx = e^{x\theta}[Ei(-\mu - \mu \theta) - Ei(-\mu)] \) [30, Eq. (3.352.1)]

Similarly, we calculate \( A_3 \) in the following: (see (34)), where we use \( \int_{x}^{x+1}(x) \, dx = -e^{x\theta}[Ei(-\mu - \mu \theta) - Ei(-\mu)] \) [30, Eq. (3.352.2)].

After we obtain \( A_2 \) and \( A_3 \), the average rate of \( x \) can be calculated with \( R = A_2 + A_W \) and Theorem 2 can thus be obtained.
8.3 Proof of Corollary 1

When \( 0 < a_i < (f_2, r)(y_1 + y_2 + y_1, y_1, t_1, t_1) \), we have the outage probability shown in (19), thus the derivation of \( P_{\text{out}} \) with respect to the power allocation coefficient \( a_i \) can be obtained as

\[
\frac{\partial P_{\text{out}}}{\partial a_i} = -\frac{\rho_i \beta_{1,2}}{\rho_i \beta_{1,2} + \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i} \frac{\gamma_i \mu e^{-\gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i} e^{-(y_1, y_1, y_1, y_1, y_1, y_1)} < 0. \quad (35)
\]

When \((y_1, r)(y_1 + y_2 + y_1, y_1, t_1, t_1) \leq a_i < (1/\gamma_i + 1)\), the outage probability is shown in (20), therefore the derivation of \( P_{\text{out}} \) in terms of \( a_i \) is

\[
\frac{\partial P_{\text{out}}}{\partial a_i} = \frac{\rho_i \beta_{1,2}}{\rho_i \beta_{1,2} + \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i} \left( \frac{\gamma_i \mu e^{-\gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i, \gamma_i} e^{-(y_1, y_1, y_1, y_1, y_1, y_1)}}{a_i - a_i, 1, 1} \right) - e^{-(y_1, y_1, y_1, y_1, y_1, y_1)} \leq 0. \quad (36)
\]

Based on (35) and (36), we can see that \( P_{\text{out}} \) monotonically decreases as \( a_i \) grows for \( 0 < a_i < (f_2, r)(y_1 + y_2 + y_1, y_1, t_1, t_1) \), and monotonously increases as \( a_i \) grows for \((y_1, r)(y_1 + y_2 + y_1, y_1, t_1, t_1) \leq a_i < (1/\gamma_i + 1)\). Therefore, the optimal power allocation parameter \( a_i \) to minimise the outage probability in the CFR-NOMA system is shown in (21) and \( a_i \) is obtained by using \( a_i + a_i = 1 \), then the proof is completed.

8.4 Proof of corollary 2

To improve Corollary 2, we assume the optimal power allocation \((a_{\ast}, a_{\ast})\) can be obtained with the condition \( R_i(\alpha_{\ast}, a_{\ast}) > Q_i \) is satisfied. Thus, we have \( R_{\text{sum}}(a_{\ast}, a_{\ast}) = R(\alpha_{\ast}, a_{\ast}) + R(\alpha_{\ast}, a_{\ast}) \) when \( R(\alpha_{\ast}, a_{\ast}) > Q_i \). Then there must exist another power allocation \((\alpha_{\ast}, a_{\ast})\), where \( \alpha_{\ast} = a_{\ast} - \epsilon \), \( a_{\ast} = a_{\ast} + \epsilon \) and \( \epsilon \) is small enough positive value, such that \( R_{\text{sum}}(\alpha_{\ast}, a_{\ast}) < R_{\text{sum}}(\alpha_{\ast}, a_{\ast}) \) will hold. However, we obtain \( R_{\text{sum}}(\alpha_{\ast}, a_{\ast}) > R_{\text{sum}}(\alpha_{\ast}, a_{\ast}) \) in the following.

According to (2), (6), (7) and (23), we can obtain
\[
\Delta = R_{\text{sum}}(a'_w, a'_s) - R_{\text{sum}}(a''_w, a''_s)
\]
\[
= R(a'_w, a'_s) + R(a''_w, a''_s) - R(a''_w, a'_s) - R(a'_w, a''_s)
\]
\[
= \log \left[ 1 + \min \left( \frac{\lambda_1 \rho_1}{\lambda_2 \rho_2 + 1}, \frac{1 + \lambda_1 \rho_1}{\lambda_2 \rho_2 + 1} \right) \right]
\]
\[
- \log \left[ 1 + \min \left( \frac{\lambda_2 \rho_1}{\lambda_2 \rho_2 + 1}, \frac{1 + \lambda_2 \rho_1}{\lambda_2 \rho_2 + 1} \right) \right]
\]
\[
+ \log(1 + a'_w \rho_1 + a'_s \rho_2) - \log(1 + a''_w \rho_2 + a''_s \rho_2) .
\]

When \( \gamma_{1,2} \leq \gamma_{\text{MRC}} \), (37) can be further expressed as
\[
\Delta = \log \left( 1 + \frac{\lambda_1 \rho_1 + (a'_w \rho_1 + a'_s \rho_2 + 1)(a'_w \rho_1 + a'_s \rho_2)}{1 + \lambda_2 \rho_2 + a'_w \lambda_1 \rho_1 \rho_2} \right) > 0.
\] (38)

When \( \gamma_{1,2} > \gamma_{\text{MRC}} \), (37) can be further expressed as
\[
\Delta = \log \left( 1 + \frac{1 + \lambda_1 \rho_1}{1 + \lambda_2 \rho_2 + \lambda_1 \rho_1 + a'_w \lambda_1 \rho_1 \rho_2} \right) > 0.
\] (39)

From (38) and (39), we can obtain that \( \Delta > 0 \), i.e. \( R_{\text{sum}}(a'_w, a'_s) > R_{\text{sum}}(a''_w, a''_s) \), which is contradicted with the assumption and proves Corollary 2. Here, we complete the proof of (25).