Impact of Load Balancing on Rate Coverage Performance in Millimeter Wave Cellular Heterogeneous Networks

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Abstract—We conduct novel analysis to assess the impact of load balancing on the rate coverage performance in a millimeter wave (mmWave) cellular heterogeneous network (HetNet) with a macro-tier and a micro-tier. To ensure the generality of our analysis, we adopt the Poission point process for the location of base stations (BSs) and user equipments, the line-of-sight ball model for mmWave links, the sectored antenna model for key antenna array characteristics, and Nakagami-$m$ fading for wireless channels. We first analyze the loads of macro- and micro-tier BSs, based on which we derive a new expression for the rate coverage probability of the network. Through numerical results, we demonstrate the correctness of our analysis. In addition, we thoroughly examine the impact of load balancing and various network parameters on the rate coverage probability in various scenarios, offering valuable guidelines into the design of practical mmWave HetNets.

I. INTRODUCTION

Looking 3–5 years ahead, the fifth generation (5G) cellular networks will provide ultra-high-quality wireless service, such as up-to-gigabits per second throughput, towards a large number of mobile users. To bring such networks into reality, millimeter wave (mmWave) and network densification have been acknowledged as two highly promising techniques and received substantial interests from academia and industry [1, 2]. Due to their potentials, the joint exploitation of mmWave and network densification has emerged as a quickly advancing research direction, the outcomes of which will help to alleviate the spectrum shortage problem facing global cellular providers and meet the rapidly growing data rate requirement.

To fully reap the benefits of mmWave communications, antenna arrays are employed at base stations (BSs) for performing directional beamforming [3–5]. Thanks to the small wavelength of the mmWave band, it is possible to pack multiple antenna elements into the limited space at mmWave BSs, creating very narrow beams to provide a very high gain. Against this background, numerous research efforts have been devoted to evaluating mmWave wireless networks, e.g., [6–9].

As a highly effective mechanism to densify wireless cellular networks, the heterogeneous network (HetNet) architecture deploys both macro BSs and low-power BSs to serve user equipments (UEs). As pointed out by the 3rd generation partnership project, a major issue in the HetNet is that macro BSs are often heavily loaded, while low-power BSs are always lightly loaded [10]. This load disparity inevitably leads to suboptimal resource allocation across the network; therefore, a large number of UEs may be associated with one macro BS but experience poor data rate. To increase the load of low-power BSs and strike a load balance between macro BSs and low-power BSs, an association bias factor needs to be added to increase the possibility that UEs are associated with low-power BSs [10, 11]. This method, referred to as the cell range extension (CRE) [10, 11], offloads more UEs to low-power BSs, expands the association areas of low-power BSs, and enables the network to better allocate its resource among UEs.

We note that the previous studies have investigated the CRE only in sub-6 GHz cellular HetNets, e.g., [12, 13]. However, these results cannot be easily extended to mmWave networks, due to fundamental differences between mmWave HetNets and conventional HetNets. First, highly directional beams are applied in mmWave cellular HetNets, which significantly changes the interference behavior across the network. Second, the blocking effect in mmWave cellular HetNets reduces the received signal strength by tens of dBs, therefore cannot be ignored in mmWave cellular HetNets. Due to these reasons, new studies are needed to evaluate and optimize the impact of CRE on the performance of mmWave cellular HetNets, which motivates our work. This impact has not been examined in the existing studies on mmWave networks, e.g., [3, 6].

In this paper, we examine the impact of load balancing in a two-tier generalized mmWave cellular HetNet where the macro BSs in the macro-tier and the low-power BSs in the micro-tier co-exist to serve UEs. The generality of our considered network lies in the use of the Poisson point process (PPP) to model the location of BSs and UEs, the use of the LoS ball model to characterize the probability of a communications link being LoS, the use of the sectored antenna model at BSs to capture key antenna array properties, and the use of versatile Nakagami-$m$ fading to model the wireless channel. We assume that a bias factor is used in the network to offload UEs to low-power BSs. For this network, we first analyze the loads of macro BSs and low-power BSs. Based on such analysis, we derive a new expression for the rate coverage probability of the network. Using numerical results, we demonstrate the accuracy of our analytical expressions and show how the rate coverage probability is affected by the bias factor in various scenarios, offering valuable guidelines into the design of practical mmWave HetNets.

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scenarios. We further comprehensively examine the impact of other network parameters on the rate coverage probability.

II. SYSTEM MODEL

We consider a mmWave cellular HetNet, as illustrated in Fig. 1, which consists of a macro-tier and a micro-tier. The macro-tier consists of macro BSs and the micro-tier consists of low-power BSs. We model the macro BS location and the low-power BS location as two independent homogeneous PPPs, denoted by \( \Phi_{all,m} \) with the intensity \( \lambda_{all,m} \) on \( \mathbb{R}^2 \) and \( \Phi_{all,s} \) with the intensity \( \lambda_{all,s} \) on \( \mathbb{R}^2 \), respectively. We also model the UE location as another independent PPP, denoted by \( \Phi_u \) with the intensity \( \lambda_u \) on \( \mathbb{R}^2 \). In this work, we randomly select one UE and refer to it as the typical UE (i.e., UE A in Fig. 1). Then we establish a polar coordinate system with the typical UE at the pole. Based on the Slivnyak theorem \([11]\), the conclusions drawn for the typical UE can be extended to other UEs.

To build a systematic performance study of the considered mmWave cellular HetNet, we need to incorporate the unique characteristics of mmWave communications into the network, as follows:

1) Blockage Effect: We adopt an accurate yet simple LoS ball model in the network, due to its high suitability for the system-level analysis of mmWave networks \([3–5, 7, 14, 15]\). In the LoS ball model, the probability of a communications link being LoS is a function of the distance between the BS and the UE, \( r \). In this work, we use \( P_{\text{LoS}}(r) \) to represent this probability, where \( \xi = m \) for the macro-tier and \( \xi = s \) for the micro-tier. Mathematically, we express \( P_{\text{LoS}}(r) \) as

\[
P_{\text{LoS}}(r) = \begin{cases} \omega_m, & \text{if } 0 < r < \mu_m, \\ 0, & \text{otherwise}, \end{cases}
\]

where \( 0 \leq \omega_m \leq 1 \) is the value of the probability and \( \mu_m > 0 \) is depicted in Fig. 1. In this work, we only consider LoS BSs, due to the extremely high outdoor penetration loss of mmWave propagation \([2, 16]\). According to \([17, \text{Prop. (1.3.5)}]\), the LoS macro BS location observed at the typical UE follows the PPP \( \Phi_m \) with the intensity \( \lambda_{all,m} \omega_m \) within the circular area \( B(0, \mu_m) \). Moreover, the LoS low-power BS location observed at the typical UE follows the PPP \( \Phi_s \) with the intensity \( \lambda_{all,s} \omega_s \) within the circular area \( B(0, \mu_s) \). These two PPPs are illustrated in Fig. 1. For the simplicity of presentation, we define \( \lambda_m \triangleq \lambda_{all,m} \omega_m \) and \( \lambda_s \triangleq \lambda_{all,s} \omega_s \).

2) Directional Beamforming: We assume that each BS is equipped with an antenna array to perform directional beamforming. We also assume that each UE is equipped with a single omnidirectional antenna. As adopted in \([3–5]\), the actual antenna array gain pattern of BSs is approximated by a sectored antenna model, which captures the key antenna array characteristics including the beamwidth, the main lobe gain, and the side lobe gain. Mathematically, this pattern is expressed as

\[
G_\xi(\theta) = \begin{cases} G_{\text{max},\xi}, & \text{if } |\theta| \leq \theta_{\xi,\text{max}}, \\ G_{\text{min},\xi}, & \text{otherwise}, \end{cases}
\]

where \( G_{\text{max},\xi} \) and \( G_{\text{min},\xi} \) are the main lobe gain and the side lobe gain, respectively. \( \theta \) is the angle off the boresight direction, and \( \theta_{\xi,\text{max}} \) is the beamwidth of the BS.

3) Fading Channel Model: In this work, we adopt independent Nakagami-\( M \) fading for each link to model the small scale fading \( h \), where \( M \) is the fading parameter. This adoption arises from the fact that Nakagami-\( M \) fading is a generalized model which particularly encompasses Rician fading, a typically used model for LOS scenarios, as a special case \([18, 19]\). Against this background, Nakagami-\( M \) fading has recently been adopted in the performance analysis of mmWave cellular networks \([3, 7]\).

We define \( h \triangleq |h|^2 \) as the small scale fading power gain. Therefore, \( h \) follows the Gamma distribution with the shape parameter \( M \) and the scale parameter \( 1/M \). It follows that the cumulative distribution function (CDF) of \( h \) is given by

\[
F_h(x) = 1 - e^{-x/M} \sum_{k=0}^{M-1} \frac{(Mx)^k}{k!}.
\]

where \( M \) is assumed to be a positive integer. Accordingly, the moment generation function (MGF) of \( h \) is given by

\[
M_{h}(s) = \left(1 - \frac{s}{M} \right)^{-M}.
\]

We assume that all macro BSs have the same transmit power \( P_m \) and all low-power BSs have the same transmit power \( P_s \). Thus, the received power at the typical UE from a BS is given by \( P = P_\xi G_\xi(\theta) r^{-\alpha} h \), where \( r \) is the distance between the BS and the typical UE and \( \alpha \) is the path loss exponent. In this work, the maximum biased received signal strength (RSS) UE association algorithm \([12]\) is adopted. Therefore, the typical UE chooses the nearest LoS macro BS as its serving BS if

\[
P_m r_{\text{min},m}^{-\alpha} G_{\text{max},m} > A_s P_s r_{\text{min},s}^{-\alpha} G_{\text{max},s} \Rightarrow r_{\text{min},s} > r_{\text{min},m},
\]

where \( A_s \) is the bias factor, \( r_m \) is the distance between a LoS macro BS and the typical UE, \( r_s \) is the distance between a
LoS low-power BS and the typical UE, \( r_{\min,m} = \min\{r_m\} \), \( r_{\min,s} = \min\{r_s\} \), and \( \rho \triangleq \frac{(P_m G_{\text{max,m}}/A_s P_s G_{\text{max,s}})^{-1/\alpha}}{} \). If \( r_{\min,s} \leq \rho_{\min,m} \), the typical UE chooses the nearest LoS low-power BS as its serving BS. As per the UE association algorithm, one BS can be chosen by multiple UEs at the same time. To avoid inter-user interference, we adopt the time-division multiple access (TDMA) scheme due to its popularity in the existing mmWave standards such as IEEE 802.11ad [20], IEEE 802.15.3c [21], and ECMA-387 [22].

III. LOAD AND RATE COVERAGE PROBABILITY ANALYSIS

In this paper, we aim to characterize the relationship between the bias factor, \( A_s \), and the rate coverage probability, \( P_c \). This characterization is completed in two steps. First, we derive the load of a typical macro BS and the load of a typical low-power BS, where the load of a BS is defined as the mean number of the UEs associated with this BS. Second, we analyze the rate coverage probability at the typical UE.

A. Load Analysis

According to the Slivnyak theorem [11], the load of a macro BS is given by \( \lambda_u P_m / \lambda_{all,m} \), where \( P_m \) is the probability that a UE is associated with a macro BS. Similarly, the load of a low-power BS is given by \( \lambda_u P_s / \lambda_{all,s} \), where \( P_s \) is the probability that a UE is associated with a low-power BS. For the convenience of presentation, we define \( L_m \triangleq \lambda_u P_m / \lambda_{all,m} \) and \( L_s \triangleq \lambda_u P_s / \lambda_{all,s} \). To evaluate \( L_m \) and \( L_s \), we need to derive \( P_m \) and \( P_s \).

To commence our derivation, we define \( B_m \triangleq \Pr(\Phi_m \neq \emptyset) \) and \( B_s \triangleq \Pr(\Phi_s \neq \emptyset) \). Based on [23, Eq. (2.15)], we find that

\[
B_m = 1 - e^{-\lambda_m \mu^2 m} \quad \text{and} \quad B_s = 1 - e^{-\lambda_s \mu^2 s}.
\]

We also find that \( B_m \) and \( B_s \) are not necessarily equal to 1, which implies that the typical UE is possible to find no LoS macro BSs or LoS low-power BSs to associate. Thus, we define five possible scenarios of the considered mmWave cellular HetNet, as follows:

- Scenario 1: \( \Phi_m = \emptyset \) and \( \Phi_s = \emptyset \).
- Scenario 2: \( \Phi_m \neq \emptyset \) and \( \Phi_s = \emptyset \).
- Scenario 3: \( \Phi_m = \emptyset \) and \( \Phi_s \neq \emptyset \).
- Scenario 4: \( \Phi_m \neq \emptyset \) and \( \Phi_s \neq \emptyset \), while the typical UE is associated with the nearest LoS macro BS.
- Scenario 5: \( \Phi_m \neq \emptyset \) and \( \Phi_s \neq \emptyset \), while the typical UE is associated with the nearest LoS low-power BS.

Given \( \Phi_k \neq \emptyset \), the CDF and the probability density function (PDF) of \( r_{\min,k} \) are given by

\[
F_{r_{\min,k}}(x) = \begin{cases} 
\frac{1-e^{-\lambda_k \pi x^2}}{B_k}, & \text{if } 0 \leq x \leq \mu_k, \\
1, & \text{if } x > \mu_k, 
\end{cases} \quad \text{(6)}
\]

and

\[
f_{r_{\min,k}}(x) = \begin{cases} 
\frac{2\lambda_k \pi x e^{-\lambda_k \pi x^2}}{B_k}, & \text{if } 0 \leq x \leq \mu_k, \\
0, & \text{if } x > \mu_k, 
\end{cases} \quad \text{(7)}
\]

respectively. Using (6) and (7), we obtain \( P_{t,m} \) as

\[
P_{t_m} = \Pr(\text{Scenario 2}) + \Pr(\text{Scenario 4}) = B_m (1 - B_s) + \Pr(\Phi_m \neq \emptyset) \Pr(\Phi_s \neq \emptyset)
\]

\[
\times \Pr \left( r_{\min,m} < \frac{r_{\min,s}}{\rho} | \Phi_m \neq \emptyset, \Phi_s \neq \emptyset \right)
\]

\[
= B_m (1 - B_s) + B_m B_s \mathbb{E}_{r_{\min,s}} \left[ F_{r_{\min,m}} \left( \frac{r_{\min,s}}{\rho} \right) \right]
\]

\[
= B_m (1 - B_s) + B_m B_s \int_0^{\mu_s} f_{r_{\min,s}}(x) F_{r_{\min,m}}(\frac{x}{\rho}) \, dx,
\]

where \( \mathbb{E}[\cdot] \) denotes expectation. Similarly, we obtain \( P_{t,s} \) as

\[
P_{t_s} = \Pr(\text{Scenario 3}) + \Pr(\text{Scenario 5}) = B_s (1 - B_m) + B_m B_s \int_0^{\mu_m} f_{r_{\min,m}}(x) F_{r_{\min,s}}(\frac{x}{\rho}) \, dx.
\]

Substituting (8) and (9) into \( L_m = \lambda_u P_m / \lambda_{all,m} \) and \( L_s = \lambda_u P_s / \lambda_{all,s} \), respectively, we obtain the final expressions for \( L_m \) and \( L_s \). Such expressions determine the loads of the macro BS and low-power BS and allow us to analyze the rate coverage probability of the network.

B. Rate Coverage Probability Analysis

In the considered mmWave cellular network with TDMA, the maximum rate at which the information can be transmitted by a macro BS or a low-power BS is

\[
R = \frac{S_\xi}{L_\xi} \log_2 (1 + \text{SINR}),
\]

where \( S_\xi \) is the spectrum resource allocated to a macro BS or a low-power BS and \( L_\xi \) is obtained in Section III-A. Relying on (10), the rate coverage probability of the network, \( P_c \), is defined as the probability that the maximum rate of the network, \( R \), is larger than the rate threshold, \( \delta \), i.e.,

\[
P_c \triangleq \Pr \left( R > \delta \right) = \frac{1}{5} \sum_{i=1}^{5} \Pr \left( R > \delta, \text{Scenario } i \right).
\]

\[
\begin{align*}
\Pr \left( R > \delta, \text{Scenario 1} \right) &= \Pr \left( R > \delta, \text{Scenario 2} \right) + \Pr \left( R > \delta, \text{Scenario 3} \right) + \Pr \left( R > \delta, \text{Scenario 4} \right) + \Pr \left( R > \delta, \text{Scenario 5} \right) \\
&= \Pr \left( \text{SINR} > \frac{2 \delta_{\min}}{3}, \text{Scenario 2} \right) + \Pr \left( \text{SINR} > \frac{2 \delta_{\min}}{3}, \text{Scenario 3} \right) + \Pr \left( \text{SINR} > \frac{2 \delta_{\min}}{3}, \text{Scenario 4} \right) + \Pr \left( \text{SINR} > \frac{2 \delta_{\min}}{3}, \text{Scenario 5} \right) \\
&= P_2 \left( \frac{2 \delta_{\min}}{3} - 1, \text{Scenario 2} \right) + P_3 \left( \frac{2 \delta_{\min}}{3} - 1, \text{Scenario 3} \right) + P_4 \left( \frac{2 \delta_{\min}}{3} - 1, \text{Scenario 4} \right) + P_5 \left( \frac{2 \delta_{\min}}{3} - 1, \text{Scenario 5} \right) \\
&= \Pr \left( \text{SINR} > \frac{2 \delta_{\min}}{3}, \text{Scenario 1} \right).
\end{align*}
\]

where the equality (i) is due to the law of total probability, the equality (ii) is due to \( \Pr \left( R > \delta, \text{Scenario } 1 \right) = 0 \), and the equality (iii) is obtained based on (10).
In order to obtain $P_e$, we next derive $P_2 (\tau)$, $P_3 (\tau)$, $P_4 (\tau)$, and $P_5 (\tau)$.

1) Analysis of $P_2 (\tau)$: In Scenario 2, the power of the received signal at the typical UE is given by $S = \kappa_m h$, where $\kappa_m = \rho_m r_{\text{min},m}^{-\alpha} G_{\text{max},m}$. The power of the interference at the typical UE is given by

$$I = I_m^* \triangleq \sum_{(r_m, \theta) \in \Phi_m / (r_m, \theta)} P_m r_m^{-\alpha} G_m (\theta) h. \quad (12)$$

Based on $I$ and $I$, we derive $P_2 (\tau)$ as

$$P_2 (\tau) = \sum \mathbb{P} (\Phi_m \neq \emptyset) \mathbb{P} (\Phi_s = \emptyset) \mathbb{P} \left( \frac{S}{I_m + \sigma^2} > \tau \bigg| \Phi_m \neq \emptyset \right)$$

$$= (1 - B_m) B_m \mathcal{E}_{r_{\text{min},m},I_m^*} \left[ \mathbb{P} \left( h > \frac{\tau (I_m^* + \sigma^2)}{\kappa_m} \bigg| \Phi_m \neq \emptyset \right) \right]$$

$$\times \sum_{k=0}^{\mathcal{M}-1} \left( \mathcal{F}_{\text{min},m} \left( I_m^* + \sigma^2 \right) \right)^k \left( \frac{1}{k!} \right) e^{-\mathcal{F}_{\text{min},m} (I_m^*)} \cdot \left( \frac{1}{k!} \right)$$

$$= (1 - B_m) B_m \int_{0}^{\mu_m} \sum_{k=0}^{\mathcal{M}-1} \mathcal{E}_{I_m^*} \left[ e^{-\mathcal{F}_{\text{min},m} (I_m^*)} \left( I_m^* + \sigma^2 \right)^k \right]$$

$$\times \left( \frac{1}{k!} \right) f_{\text{min},m} (x) dx, \quad (13)$$

where $\sigma^2$ is the noise power at the typical UE and $\mathcal{F}_{\text{min},m} = \mathcal{M} \tau / P_m G_{\text{max},m}$. In (13), the equality (a) is obtained based on the independence between $\Phi_m$ and $\Phi_s$, and the equality (b) is obtained based on the CDF of $h$ given in (3).

We now introduce the Laplace transform of $I_m^* + \sigma^2$ to obtain a simpler presentation of $P_2 (\tau)$. Specifically, the Laplace transform of $I_m^* + \sigma^2$ is given by

$$\mathcal{L}_{I_m^* + \sigma^2} (a) = \mathbb{E} \left[ e^{-a(I_m^* + \sigma^2)} \right]. \quad (14)$$

Accordingly, the $k$th derivative of $\mathcal{L}_{I_m^* + \sigma^2} (a)$ is given by

$$\mathcal{L}_{I_m^* + \sigma^2}^{(k)} (a) = \mathbb{E} \left[ e^{-a(I_m^* + \sigma^2)} (-1)^k (I_m^* + \sigma^2)^k \right]. \quad (15)$$

Substituting (15) into (13), we obtain

$$P_2 (\tau) = (1 - B_m) B_m \sum_{k=0}^{\mathcal{M}-1} \left( \frac{1}{k!} \right) (-\mathcal{F}_{\text{min},m})^k$$

$$\times \int_{0}^{\mu_m} x^k \mathcal{L}_{I_m^* + \sigma^2}^{(k)} (\mathcal{F}_{\text{min},m} x^a) f_{\text{min},m} (x) dx. \quad (16)$$

As indicated by (16), we need to find $\mathcal{L}_{I_m^* + \sigma^2} (a)$ to obtain $P_2 (\tau)$. Based on (14), we obtain $\mathcal{L}_{I_m^* + \sigma^2} (a)$ as

$$\mathcal{L}_{I_m^* + \sigma^2} (a) = e^{-\mathcal{F}_{\text{min},m} (I_m^*)} \mathcal{L}_{I_m^*} (a)$$

$$= e^{-a \mathcal{F}_{\text{min},m} (I_m^*)} f_{\text{min},m} (x) \left( 1 - e^{-h a P_m G_{\text{max},m}(\theta) - \alpha} \right) dx, \quad (17)$$

where the equality (c) is obtained based on [17, Cor. (2.3.2)], the equality (d) is obtained based on the MGF of $h$ given in (4), and the equality (e) is obtained based on the antenna gain function given in (2). In (17), we define $\Omega_m$ as

$$\Omega_m = \theta_m \left[ 1 - \left( 1 + \frac{a P_m G_{\text{max},m}}{M r^\alpha} \right)^{-\mathcal{M}} \right]$$

$$+ (2 \pi - \theta_m) \left[ 1 - \left( 1 + \frac{a P_m G_{\text{min},m}}{M r^\alpha} \right)^{-\mathcal{M}} \right]. \quad (18)$$

2) Analysis of $P_3 (\tau)$: Following the derivation procedure of $P_2 (\tau)$, we obtain $P_3 (\tau)$ as

$$P_3 (\tau) = (1 - B_m) B_m \sum_{k=0}^{\mathcal{M}-1} \left( \frac{1}{k!} \right) (-\mathcal{F}_{\text{min},m})^k \times \int_{0}^{\mu_m} x^k \mathcal{L}_{I_m^* + \sigma^2}^{(k)} (\mathcal{F}_{\text{min},m} x^a) f_{\text{min},m} (x) dx,$$

where $\mathcal{F}_{\text{min},m} = \mathcal{M} \tau / P_m G_{\text{max},m}$, and

$$\mathcal{L}_{I_m^* + \sigma^2}^{(k)} (a) = e^{-a \mathcal{F}_{\text{min},m} (I_m^*)} f_{\text{min},m} (x) dx.$$

In (20), we define $\Omega_s$ as

$$\Omega_s = \theta_s \left[ 1 - \left( 1 + \frac{a P_m G_{\text{min},s}}{M r^\alpha} \right)^{-\mathcal{M}} \right]$$

$$+ (2 \pi - \theta_s) \left[ 1 - \left( 1 + \frac{a P_m G_{\text{min},s}}{M r^\alpha} \right)^{-\mathcal{M}} \right]. \quad (21)$$

3) Analysis of $P_4 (\tau)$: In Scenario 4, the power of the received signal at the typical UE is given by $S = \kappa_m h$ and power of the interference at the typical UE is given by $I = I_m^* + I_s$, where $I_m^*$ is defined in (12) and $I_s$ is given by

$$I_s \triangleq \sum_{(r_s, \theta_s) \in \Phi_s} P_s r_s^{-\alpha} G_s (\theta) h. \quad (22)$$

Based on $S$ and $I$, we derive $P_4 (\tau)$ as

$$P_4 (\tau) = \mathbb{P} (\text{SINR} > \tau, \Phi_m \neq \emptyset, \Phi_s \neq \emptyset, r_{\text{min},m} > \rho r_{\text{min},m})$$

$$= \mathbb{P} (\text{SINR} > \tau, \Phi_m \neq \emptyset, \Phi_s \neq \emptyset, r_{\text{min},s} > \rho r_{\text{min},m}), \quad (23)$$

Following the derivation procedure of $P_2 (\tau)$, we obtain $P_4 (\tau)$ as

$$P_4 (\tau) = 2 \lambda_m \pi \sum_{k=0}^{\mathcal{M}-1} \left( \frac{1}{k!} \right) (-\mathcal{F}_{\text{min},m})^k q (\tau), \quad (24)$$

where

$$q (\tau) = \int_{0}^{\min \left( \frac{a \mu_m}{\mathcal{M}}, \mu_m \right)} \left( B_s - 1 + e^{-k^2 \pi x^2} \right) x^{k+1} \mathcal{L}_{I_m^* + I_s + \sigma^2}^{(k)} (\mathcal{F}_{\text{min},m} x^a) e^{-\mathcal{F}_{\text{min},m} (x^a) dx}, \quad (25)$$

with

$$\mathcal{L}_{I_m^* + I_s + \sigma^2}^{(k)} (a) = e^{-a \mathcal{F}_{\text{min},m} (I_m^*)} r_{\text{min},m} (x^a) r_{\text{min},s} (x^a). \quad (26)$$


\[ P_c = (1 - B_s) B_m \sum_{k=0}^{M-1} \frac{(-\varepsilon_m)^k}{k!} \int_0^{\mu_m} x^{\alpha k} \mathcal{L}_{m+\sigma^2}^{(k)}(\varepsilon_m x^\alpha) f_{\text{min},m}(x) dx + (1 - B_m) B_s \sum_{k=0}^{M-1} \frac{(-\varepsilon_s)^k}{k!} \int_0^{\mu_s} x^{\alpha k} \mathcal{L}_{s+\sigma^2}^{(k)}(\varepsilon_s x^\alpha) f_{\text{min},s}(x) dx \]  

\times \left[ \int_0^{\mu_s} x^{\alpha k} \mathcal{L}_{s+\sigma^2}^{(k)}(\varepsilon_s x^\alpha) f_{\text{min},s}(x) dx + 2\lambda_m \pi \sum_{k=0}^{M-1} \frac{(-\varepsilon_m)^k}{k!} \Phi(2^{\frac{m\mu_m}{\lambda_s}} - 1) + 2\lambda_s \pi \sum_{k=0}^{M-1} \frac{(-\varepsilon_s)^k}{k!} \Phi(2^{\frac{m\mu_s}{\lambda_s}} - 1) \right]. \tag{30} 

4) Analysis of \( P_5(\tau) \): Following the derivation procedure of \( P_3(\tau) \), we derive \( P_5(\tau) \) as

\[ P_5(\tau) = 2\lambda_s \pi \sum_{k=0}^{M-1} \frac{1}{k!} (\psi_s)^k \varpi(\tau), \tag{27} \]

where

\[ \varpi(\tau) = \int_0^{\min(\mu_s, \mu_m)} \left( B_m - 1 + e^{-\lambda_m \pi x^2} \right) x^{\alpha k+1} \times \mathcal{L}_{m+\sigma^2}^{(k)}(\psi_s x^\alpha) e^{-\lambda_s \pi x^2} dx. \tag{28} \]

with

\[ \mathcal{L}_{I_m+I_s+\sigma^2}(a) = e^{-a\sigma^2} - \lambda_m \int_0^{\mu_m} r \Omega_m dr - \lambda_s \int_0^{\mu_s} r \Omega_s dr. \tag{29} \]

Substituting (16), (19), (24), and (27) into (11), we obtain the exact rate coverage probability of the network, \( P_c \), as shown in (30) at the top of next page, where \( \varepsilon_m = M \left( 2^{\frac{M\mu_m}{\lambda_s}} - 1 \right) / P_m G_{\text{max},m} \) and \( \varepsilon_s = M \left( 2^{\frac{M\mu_s}{\lambda_s}} - 1 \right) / P_s G_{\text{max},s} \). This result allows us to investigate the impact of the bias factor and other network parameters on \( P_c \). We note that a closed-form expression for \( P_c \) is highly improbable to obtain, if not impossible, due to the consideration of PPPs in the network.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we present numerical results to examine the impact of load balancing and network parameters on the performance in different scenarios. Unless otherwise specified, the parameters used in this section are summarized in Table I.

We first examine the impact of the LoS low-power BS intensity, \( \lambda_s \), and the bias factor, \( A_s \), on the association probabilities, \( P_{tm} \) and \( P_{ts} \), in Fig. 2. By comparing the analytical curves with the Monte Carlo simulation points marked by ‘Δ’, we observe that our analytical expressions for \( P_{tm} \) and \( P_{ts} \), given in (8) and (9), respectively, precisely agree with simulations. This corroborates the accuracy of (8) and (9). Moreover, we observe that the limit value of \( P_{tm} \) as \( A_s \) grows large for \( \lambda_s = 10^{-3}/m^2 \) is lower than that for \( \lambda_s = 2 \times 10^{-4}/m^2 \). This can be analytically explained based on (8). Specifically, we find from (8) that \( \lim_{A_s \to \infty} P_{tm} = B_m (1 - B_s) \). When \( \lambda_s \) increases, there are more LoS low-power BSs available in the network and thus, \( B_s \) increases and \( P_{tm} \) decreases.

We next examine the impact of \( \lambda_s \) and the beamwidth of the BS, \( \theta_m \) and \( \theta_s \), on the rate coverage probability, \( P_c \) in Fig. 3. First, we demonstrate the correctness of our analytical expression for \( P_c \) by observing the exact match between the analytical curves and the Monte Carlo simulation points marked by ‘○’. Second, by comparing Case 2 with Cases 3 and 4, we find that \( P_c \) improves when the beam becomes narrower. This observation is expected, since the narrower the beam, the less interference caused by BSs. Third, by comparing Case

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**Table I: Parameters Used in Section IV**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{tm} ) &amp; ( P_{ts} )</td>
<td>10^4 &amp; 10^6 mW</td>
</tr>
<tr>
<td>( \lambda_m, \lambda_s, ) &amp; ( \lambda_u )</td>
<td>10^{-3}, 10^{-4}, &amp; 10^{-1}/m^2</td>
</tr>
<tr>
<td>( \mu_m ) &amp; ( \mu_s )</td>
<td>1000 &amp; 100m</td>
</tr>
<tr>
<td>( \omega_m ) &amp; ( \omega_s )</td>
<td>0.6 &amp; 0.5</td>
</tr>
<tr>
<td>( \theta_m ) &amp; ( \theta_s )</td>
<td>0.1 &amp; 0.2 rad</td>
</tr>
<tr>
<td>( G_{\text{max},m} ) &amp; ( G_{\text{max},s} )</td>
<td>4x10^5 &amp; 10^3</td>
</tr>
<tr>
<td>( G_{\min,m} ) &amp; ( G_{\min,s} )</td>
<td>1 &amp; 1</td>
</tr>
<tr>
<td>( A_s ) &amp; ( \alpha )</td>
<td>100 &amp; 2.2</td>
</tr>
<tr>
<td>( M ) &amp; ( \sigma^2 )</td>
<td>1 &amp; 1</td>
</tr>
<tr>
<td>( S_m ) &amp; ( S_s )</td>
<td>10^9 &amp; 10^9 Hz</td>
</tr>
</tbody>
</table>

---

**Fig. 2:** The association probabilities, \( P_{tm} \) and \( P_{ts} \), versus the bias factor, \( A_s \), for different values of \( \lambda_s \).

**Fig. 3:** The rate coverage probability, \( P_c \), versus the rate threshold, \( \delta \), for four cases: Case 1: \( \lambda_s = 10^{-3}/m^2 \), \( \theta_m = 0.1 \) rad, and \( \theta_s = 0.2 \) rad, Case 2: \( \lambda_s = 10^{-4}/m^2 \), \( \theta_m = 0.1 \) rad, and \( \theta_s = 0.2 \) rad, Case 3: \( \lambda_s = 10^{-4}/m^2 \), \( \theta_m = 0.2 \) rad, and \( \theta_s = 0.4 \) rad, and Case 4: \( \lambda_s = 10^{-4}/m^2 \), \( \theta_m = 0.5 \) rad, and \( \theta_s = 1 \) rad.
in the HetNet and address realistic yet complicated antenna patterns and mmWave channel models.

REFERENCES


Fig. 4. The rate coverage probability, $P_c$, versus the bias factor, $A_s$, for different values of $\mu_m$ and $\mu_s$ with $\delta = 10^{0.5}$ bits per second.

1 with Case 2, we find that deploying more low-power BSs profoundly improves the rate coverage performance when the beam is narrow.

Finally, we examine the impact of $\mu_m$, $\mu_s$, and $A_s$ on $P_c$ in Fig. 4. Since the LoS radii $\mu_m$ and $\mu_s$ are determined by the density of obstacles, their values indicate whether the transmission environment is urban or suburban. Specifically, large $\mu_m$ and $\mu_s$ correspond to the suburban environment, while small $\mu_m$ and $\mu_s$ correspond to the urban environment.

First, we find that $P_c$ with $A_s = 1$ is larger than $P_c$ with $A_s = 1$, which confirms that the rate coverage performance is improved when offloading more UEs to low-power BSs in both urban and suburban environments. Second, when $\mu_m = 200$ m and $\mu_s = 20$ m, $P_c$ is slightly improved as $A_s$ grows. Differently, for other two cases, $P_c$ is significantly improved as $A_s$ grows. This indicates that the effect of CRE is more profound in the suburban environment than in the urban environment.

Third, when $\mu_m = 1000$ m and $\mu_s = 100$ m, we find that $P_c$ first increases and then decreases when $A_s$ increases. This implies the existence of the optimal value of $A_s$ which maximizes $P_c$. Particularly, the optimal $A_s$ can be numerically found using our analysis.